# Modeling the distribution of patches with shift invariance: an application to SAR image restoration

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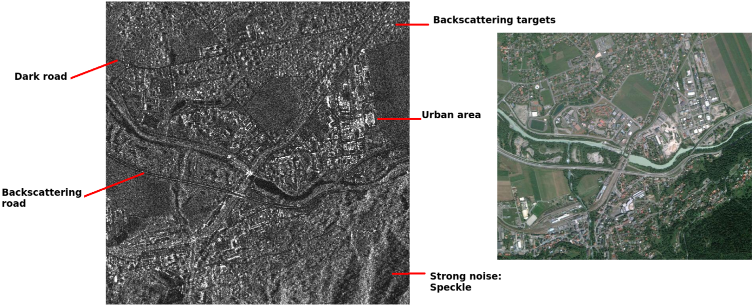
# Some challenges in SAR (Synthetic Aperture Radar) imagery




A SAR image and the optical corresponding image

- **Type of noise:** multiplicative (gamma distribution for intensity images).

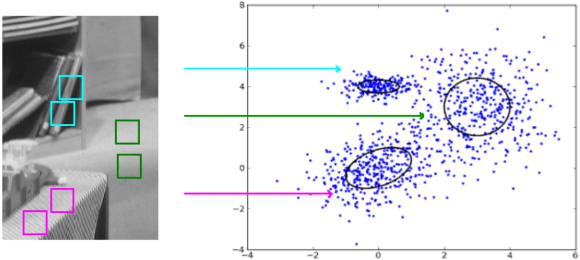
# Objectives of this work


Patches in a SAR image

- Elaborate models to learn the distribution of patches extracted from SAR images.
- Introduce shift-invariance properties in these models.
- Apply them to SAR image restoration.

# Outline

## Introduction to GMM



A patch $\boldsymbol{x}_i$ of size $8 \times 8$ from a natural image $\boldsymbol{x}$ is well modeled by a GMM:

$$\mathrm{p}(\boldsymbol{x}_i) = \sum_{k=1}^{K} w_k n_k \exp\{-\tfrac{1}{2}(\boldsymbol{x}_i - \boldsymbol{\mu}_k)^t \boldsymbol{\Sigma}_k^{-1}(\boldsymbol{x}_i - \boldsymbol{\mu}_k)\} = \sum_{k=1}^{K} \mathcal{N}(\boldsymbol{x}_i;\ \mathcal{M}_k) \qquad (1)$$

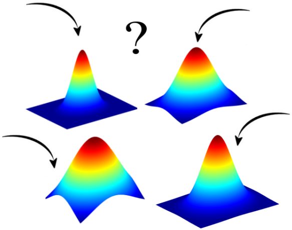
where:

- $\boldsymbol{\mu}_k$ and $\boldsymbol{\Sigma}_k$ are the mean and the covariance matrix of the $k$-th model $\mathcal{M}_k$ resp.;
- $w_k$ the $k$-th mixweight;
- $n_k = \det(2\pi\boldsymbol{\Sigma}_k)^{-1/2}$ is a normalization constant.

# How do GMM approaches work for a denoising task?

Reconstruct an image such that each of its patches is:

- close to its noisy version
- most relevant under a GMM prior.




$$p(\boldsymbol{x}_i) \approx \max_k \mathcal{N}(\boldsymbol{x}_i;\ \mathcal{M}_k) = \mathcal{N}_{k_i}(\boldsymbol{x}_i;\ \mathcal{M}_k) \qquad (2)$$

**Some algorithms based on this assumption:**

- The PLE (Piecewise Linear Estimator) [Yu et al., 2012]:
- The EPLL (Expected Patch Log Likelihood) [Zoran and Weiss, 2011].

## Main principle of the EPLL

### MAP estimate of the image

$$\min_{\boldsymbol{x}} \frac{\lambda}{2}||\boldsymbol{x} - \boldsymbol{y}||_2^2 - \log \prod_{i=1}^{N} \mathcal{N}_{k_i}(\boldsymbol{x}_i;\ \mathcal{M}_k) \quad (3)$$

With:

- $\boldsymbol{y}$ the corrupted image and $\boldsymbol{x}$ the reconstruction
- $\lambda > 0$ a parameter
- $N$ the number of patches

## Main principle of the EPLL

$$\min_{\boldsymbol{x}} \frac{\lambda}{2}||\boldsymbol{x}-\boldsymbol{y}||_2^2 - \log \prod_{i=1}^{N} \mathcal{N}_{k_i}(\boldsymbol{x}_i;\ \mathcal{M}_k) \quad (3)$$

Can be solved by the EPLL with the half quadratic splitting method:

$$\min_{\boldsymbol{x},\boldsymbol{z}_i} \frac{\lambda}{2}||\boldsymbol{x}-\boldsymbol{y}||_2^2 + \sum_i \frac{\beta}{2}||\boldsymbol{x}_i-\boldsymbol{z}_i||_2^2 - \log\ \mathcal{N}_{k_i}(\boldsymbol{z}_i;\ \mathcal{M}_k) \qquad (4)$$

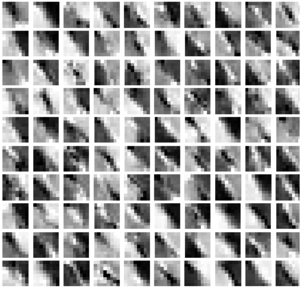
- Each $\boldsymbol{z}_i$ is an auxiliary variable associated to $\boldsymbol{x}_i$.
- $\beta$ tunes the difference between $\boldsymbol{x}_i$ and $\boldsymbol{z}_i$.
- With $\beta$ fixed, (4) can be solved alternatively for $\boldsymbol{x}$ and $\boldsymbol{z}_i$ and boil down to applying a Wiener filter.
- Their prior: a GMM (200 components) learned over $10^6$ patches extracted from natural images.

# Outline

# Is the EPLL shift-invariant?

One gaussian should explain as well all shifted versions of a same pattern inside a patch.

Edge image



Patches generated by the best gaussian

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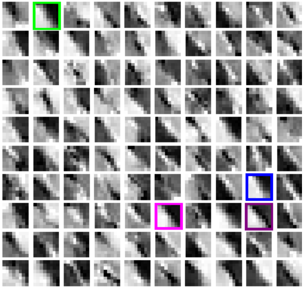
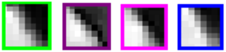


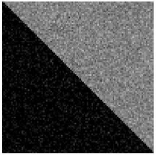
Patches generated by the best gaussian

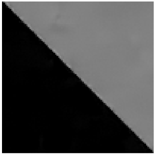


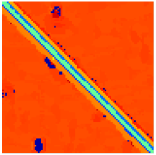
Many shifts are encoded by one gaussian ... But not all of them!

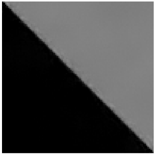
# What Gaussian Models (GM) are actually used?


Original

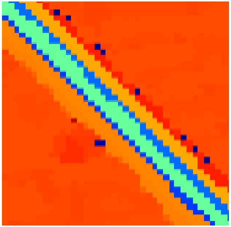

Noisy, $\sigma = 20$

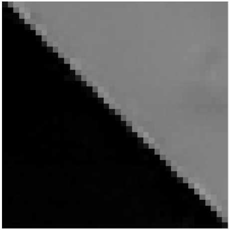

Denoised with 200 GM

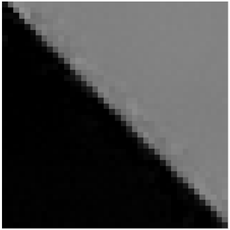

200 GM map


Denoised with 2 best GM

Zooms:


200 GM map


Denoised with 200 GM


With 2 GM

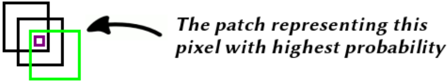
# Outline

## Introducing the shift-invariance

### The proposed prior

$$\prod_i \max_{j \in \mathcal{V}(i)} \mathcal{N}_{k_j}(\boldsymbol{x}_j;\ \mathcal{M}_k) \tag{5}$$

where $\mathcal{V}(i)$ denotes the indices of the $i$-th patch neighborhood.

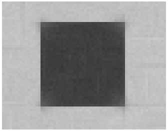


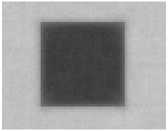

### The optimization problem adapted to shift-invariance

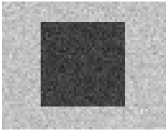
$$\min_{\boldsymbol{x}} \frac{\lambda}{2}\|\boldsymbol{x}-\boldsymbol{y}\|^2 + \sum_i \min_{j \in \mathcal{V}(i), \boldsymbol{z}_i} \left\{ \frac{\beta}{2}\|\boldsymbol{x}_j - \boldsymbol{z}_i\|^2 - \log\ \mathcal{N}_{k_j}(\boldsymbol{z}_j;\ \mathcal{M}_k) \right\} \tag{6}$$

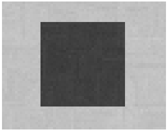
# Results on a synthetic image

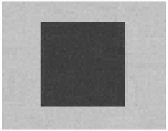

(a) Original


(b)


(c)


(d) Noisy


(e) Obtained with 10 GM

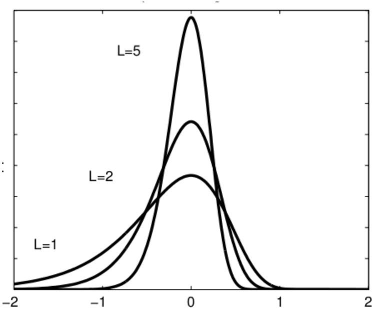

(f) Obtained with 2 GM

- (b-c) Denoised without shift-invariance
- (e-f) Denoised with shift-invariance

# Outline

## Adaptation to despeckling

- Apply a log-transformation to the noisy image.
- The square difference is no longer the best choice as a data term
  $\rightarrow$ Fisher-Tippett data-fidelity term.




Fisher-Tippett density with different parameters.

## Adaptation to despeckling

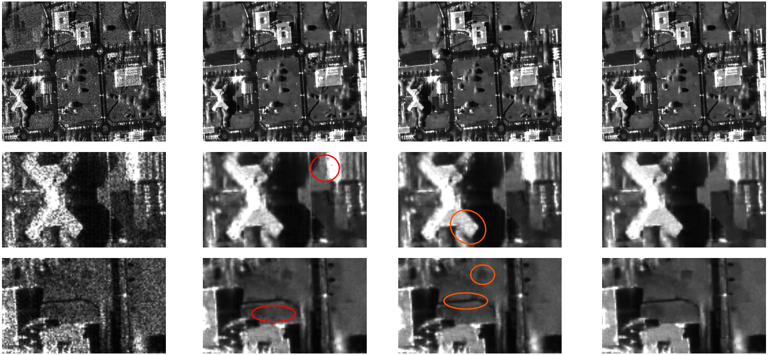
### The whole optimization problem

$$\min_{\boldsymbol{x}} \lambda \sum_{i=1}^{N} \left(e^{y_i - x_i} + x_i - y_i\right) + \min_{j \in \mathcal{V}(i), \boldsymbol{z}_i} \left\{ \frac{\beta}{2} \|\boldsymbol{x}_j - \boldsymbol{z}_i\|^2 - \log \mathcal{N}_{k_j}(\boldsymbol{z}_j;\ \mathcal{M}_k) \right\} \tag{7}$$

where $x_i$ is the $i$-th pixel of $\boldsymbol{x}$.

- Optimization along $\boldsymbol{x}$ performed with a Newton algorithm.
- Same optimization scheme as previously along $\boldsymbol{z}$.

# Results on SAR data



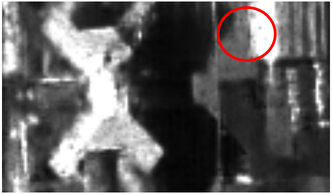
(a) 2-looks images of Toulouse

(b) log-EPLL

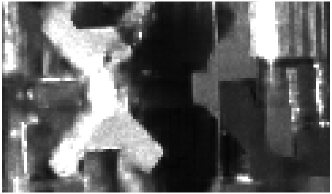
(c) FT-EPLL

(d) Shift Invariant FT-EPLL

# Zooms of the results

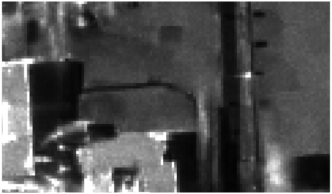


log-EPLL



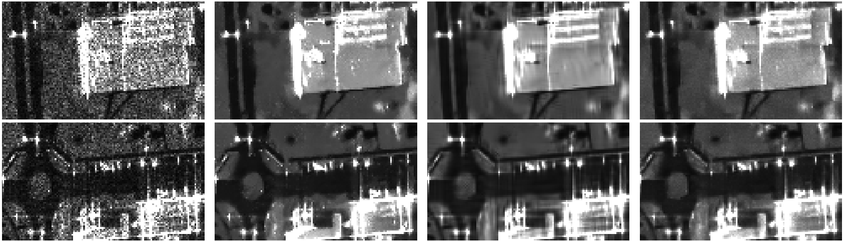
Proposed method

Fisher-Tippett-EPLL



Proposed method

# Comparisons with state of the art despeckling methods


(a) 2-looks images (b) NL-SAR filter [1] (c) BM3D filter on the log image [2] (d) Our approach

1- C.-A. Deledalle et al., NL-SAR: a unified Non-Local framework for resolution-preserving Pol In SAR denoising, 2013

2- M. Makitalo et al., Denoising of single-look SAR images based on variance stabilization and nonlocal filters, 2010

## Contributions

- Study on the information encoded by Gaussian models in a simple case.
- Introduction of the shift-invariance property in the EPLL algorithm.
- Adaptation to SAR image restoration.

## Perspectives

- Learn a GMM prior using patches extracted from nearly-noiseless SAR images.
- Is it possible to include shift-invariance in this learning?
- Extend the EPLL to SAR image classification.

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Thank you for your attention.
Any questions?

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# References I

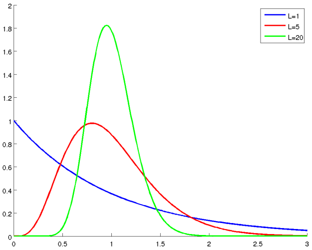

Yu, G., Sapiro, G., and Mallat, S. (2012).
Solving Inverse Problems With Piecewise Linear Estimators: From Gaussian Mixture Models to Structured Sparsity.
*IEEE Transactions on Image Processing*, 21:2481–2499.

Zoran, D. and Weiss, Y. (2011).
From Learning Models of Natural Image Patches to Whole Image Restoration.
*ICCV*.

# Heavy-tailed gamma distribution

$$p_{\mathcal{G}}(y_j|x_j) = \frac{L^L y_j^{L-1}}{x_j^L \Gamma(L)} \exp\left(-L\frac{y_j}{x_j}\right) \quad (8)$$

where $L$ is the number of looks and $\Gamma$ is the gamma function. We have: $\mathrm{Var}(y_j) = \mathbb{E}(y_j)^2/L = x_j^2/L$.




Gamma distribution with different values of $L$

# Optimization with the Fisher-Tippett data term

We can write:

$$\sum_i \|\boldsymbol{P}_{j_i^\star}\boldsymbol{x} - \boldsymbol{z}^i\|^2 = \sum_i c_i (x_i - \bar{z}_i)^2$$

where $j_i^\star$ is the index of the best-representing patch covering the pixel $i$.
Hence,

$$\sum_{i=1}^{N} \left[ \lambda(e^{y_i - x_i} + x_i - y_i) + \frac{\beta}{2} c_i (x_i - \bar{z}_i)^2 \right] \tag{9}$$

Iterative Newton-scheme for $x_i$:

$$x_i^{(t+1)} = x_i^{(t)} - \frac{\lambda(1 - e^{y_i - x_i^{(t)}}) + \beta c_i (x_i^{(t)} - \bar{z}_i))}{\lambda e^{y_i - x_i^{(t)}} + \beta c_i} \tag{10}$$

# Denoising the square image with the best Zoran and Weiss Gaussian model