Patch-based SAR image classification: the potential of modeling the statistical distribution of patches with Gaussian Mixtures

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July 29, 2015
Some challenges in SAR (Synthetic Aperture Radar) imagery

A SAR image and the optical corresponding image
Different priors

- Markov Random Fields.
- Non local.
- Gaussian Mixture Models (GMM).
- ...
Different priors

- Markov Random Fields.
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- ...
A patch $x_i$ from a natural image $x$ is well modeled by a GMM:

$$p(x_i) = \sum_{k=1}^{K} w_k n_k \exp\left\{ -\frac{1}{2} (x_i - \mu_k)^T \Sigma_k^{-1} (x_i - \mu_k) \right\} = \sum_{k=1}^{K} \mathcal{N}(x_i; \mathbf{a}_k)$$

where:

- $\mu_k$ and $\Sigma_k$ are the mean and the covariance matrix of the $k$-th model $\mathcal{M}_k$ resp.;
- $w_k$ the $k$-th mixweight;
- $n_k = \det(2\pi \Sigma_k)^{-1/2}$ is a normalization constant.
- $\mathbf{a}_k = \{\mu_k, \Sigma_k, w_k\}$ is the $k$-th Gaussian Model.
How do GMM approaches work in practice?

Reconstruct an image such that each of its patches is:

- close to its noisy version
- most relevant under a GMM prior.

Find the best GM for each patch

\[ p(x_i) \approx \max_k \mathcal{N}(x_i; a_k) \]

One of the algorithms based on this assumption:

- The EPLL (Expected Patch Log Likelihood) [Zoran and Weiss, 2011].
- The authors learnt a zero-mean 200 components GMM on a huge basis of centered 8 × 8 patches extracted from natural images.
Previous work:

- **Previous work:** Adapation of the EPLL to SAR image restauration [Tabti et al., 2014].
- **By product:** Map of the best Gaussian Model (GM) representing each pixel of the denoised image.

Idea:

Use these maps to develop a supervised classification method.
Some single SAR image classification methods

- Methods based on machine learning techniques:
  - SVM [Lardeux et al., 2009]
  - Random forests [Yang et al., 2009]

- Methods modeling texture and the amplitude of the SAR image:
  - For the texture
    - Gray Level Co-occurrence Matrices [Voisin et al., 2010]
    - Auto Regressive model [Kayabol and Zerubia, 2013]
  - For the amplitude
    - Nakagami distribution [Kayabol and Zerubia, 2013]
    - Fisher distribution [Voisin et al., 2010]

→ need for regularization, eg: Markovian models.
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**Outline**

**Context**
- Introduction to GMM
- Previous work on a patch-based SAR image restauration method
- Some single SAR image classification methods

**Proposed classification framework**
- Training phase
- Test phase

**Results**

**Conclusions**
Training phase

Learning the features

- Training set: 8 amplitude-calibrated TerraSAR-X images (sizes between $1000 \times 1000$ and $2048 \times 2048$).
- The labels are: Urban zones, high vegetation, homogeneous areas (water and low vegetation).
- A second feature is obtained the same way: after quantization of the images in 10 levels → occurrence frequency of each gray level conditionnaly to the label.
Outline

Context
  Introduction to GMM
  Previous work on a patch-based SAR image restoration method
  Some single SAR image classification methods

Proposed classification framework
  Training phase
  Test phase

Results

Conclusions
Test phase

Proposed classification framework

Results

Conclusions

Noisy image to classify

Denoised

GM map + quantized image

Probability for the $i$-th pixel to belong to the label $\mathcal{L}$

$$\log p(a^i = a_k, \nu^i = \nu_q | \mathcal{L}) = \log p(a_k | \mathcal{L}) + \log p(\nu_q | \mathcal{L})$$

- $a^i$ is the $i$-th pixel corresponding GM.
- $\nu^i$ is the $i$-th pixel corresponding radiometry.
Maximizing the overall a posteriori probability

Find the label minimizing:

\[- \log \sum_i p(a^i, \nu^i | \mathcal{L}^i) + \beta \sum_{i \sim j} \delta(\mathcal{L}^i, \mathcal{L}^j)\]

- where: $\beta > 0$, $i \sim j$ are the indices of neighbor pixels in an eight-connectivity system.
- $\mathcal{L}^i$ is the $i$-th pixel label, $\delta(\mathcal{L}^i, \mathcal{L}^j) = 1$ if $\mathcal{L}^i = \mathcal{L}^j$, 0 otherwise.
- Solutions obtained iteratively obtained by graph-cuts with the $\alpha - \beta$ swap strategy [Boykov et al., 2001].
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Legend: Urban, High vegetation, Low vegetation and water, Unlabeled

91.71%  
86.71%  
75.37%
Contributions

We proposed:

▶ A new supervised single amplitude SAR image classification algorithm.
▶ A simple framework showing the potential of a GMM-based feature.

Perspectives

▶ Enlarge the training basis.
▶ Use a GMM learnt on SAR images instead of the GMM learnt by [Zoran and Weiss, 2011].
▶ Use a more sophisticated learning technique, e.g.: SVM,...
▶ Compare our results with state of the art methods.
Thank you for your attention.
Any questions?
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References

Fast approximate energy minimization via graph cuts.

Unsupervised amplitude and texture classification of SAR images with multinomial latent model.

Support vector machine for multifrequency SAR polarimetric data classification.

Modeling the distribution of patches with shift-invariance: application to SAR image restoration.
*ICIP*.

Classification of very high resolution SAR images of urban areas by dictionary-based mixture models, copulas and Markov random fields using textural features.
In Lorenzo Bruzzone, editor, *SPIE Remote Sensing*, volume 7830, Toulouse, France. SPIE.

Supervised land-cover classification of TerraSAR-X imagery over urban areas using extremely randomized clustering forests.

From learning models of natural image patches to whole image restoration.
*ICCV*. 
Histograms representing the probability of occurrence of each Gaussian Model for each label. Each histogram varies from one label to another and this allows the GMM-based feature to discriminate the different labels.